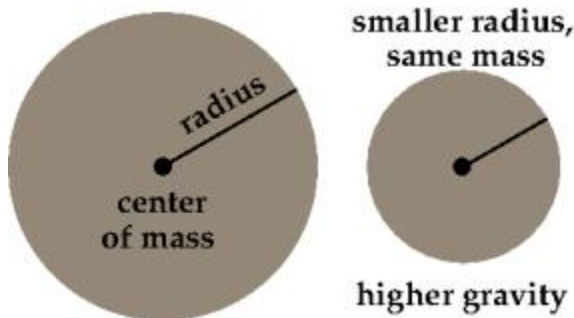


- b. The movie shows how all joy and pleasure must be earned, not given or received. As an example, the movie shows the joy that the Little Prince and the pilot feel when they taste the water from the well. It's sweetness comes from their journey under the stars and the work of the pilot's arms. What does this say about their friendship?
- c. The Little Prince finds the answers to his questions about what is important in life. When he realizes his love for the flower, he accepts that he must return to his star to care for the rose. How does he plan to do this?
- d. Why does the narrator assume the Little Prince returns home?
6. The little prince's home is the asteroid B612, about the size of a house, and yet he walks around and tends to his rose and his volcanoes just as if an asteroid that size had any significant gravitation. Let's look at the amount of the gravity force acting on the Little Prince. We'll make the following assumptions: B612 has a radius of about 2.0 meters and is made mostly of iron with a density of 13,000 kg/m³. The little prince, being just a wee lad, has a mass of 40 kg.
- a. Find the volume of the B612. Assume that it is spherical. $V = \frac{4}{3}\pi r^3$
- b. Find the approximate mass of the B612. Use: $m_1 = DV$
- c. Calculate the force of gravity between the asteroid B612 and the little prince. Use Newton's law of gravitation. $F = G \frac{m_1 m_2}{r^2}$, $G = 6.67 \times 10^{-11}$
- d. Calculate the escape velocity of B612. Use
- $$v_e = \sqrt{\frac{2GM}{r}}, \quad \text{where } M = m_1 \text{ for this case.}$$
- e. For comparison.....How long would it take to move an object 1 meter at the escape velocity magnitude. Use $v = d/t$

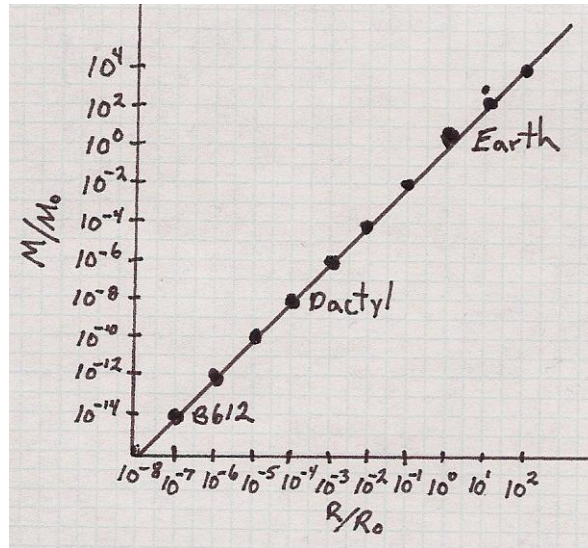
- f. Would it be possible to keep an atmosphere anchored to B612? Why? [Explain by using the speed of a molecule in a gas.]
- g. Would the little Prince's blood boil? Why?
- h. Would you agree that the author of "The Little Prince", in an attempt to present his idealistic philosophical views, is unwilling, or too squeamish, to face the harsh realities and the brutal conditions of life (or lack thereof) on inhospitable chunks of space rocks?
7. What would it take for an asteroid the size of a house to have Earthlike gravity, and what life would be like on such a tiny world? The relationship of gravity to size: if two objects have the same mass, but one is smaller than the other, the smaller object will have correspondingly higher surface gravity. Note that if the object (planet or whatever) is spherical, the center of mass is at the same location as the physical center.



Gravity in Isaac Newton's theory is a force acting between two masses. (General relativity, the more modern theory of gravity that uses the geometry of spacetime instead of a force extending through space, gives the same results when gravity is weak. More on this later.) The farther apart the two masses, the weaker the force: doubling the distance results in a force 1/4 as strong. However, there's a quirk—gravity acts as though each mass were concentrated at a point, so when you stand on the surface of Earth, the "separation" is the radius of the planet. This means if you are at the top of Everest, the force of gravity is slightly weaker than if you are standing in Death Valley. Similarly, since Earth isn't perfectly spherical, gravity is slightly stronger at the poles than it is at the equator. (To make things even more complicated, Earth's rotation also influences the force we feel!) However, we'll assume Earth and the asteroids are spherical, just for simplicity.

Because gravity gets weaker with larger distances, if we want an asteroid the size of a house with Earthlike gravity, it will need to be very massive: far more massive than a typical asteroid can be. Though we don't have a measured mass for Dactyl, if it's a fairly typical composition and density (say 2.5 times the density of water, which is slightly less than half Earth's density), its mass will be around 5×10^{12} kilograms (roughly 5 million tons). While it may *sound* like it's

extremely massive, that's about 9×10^{-13} times Earth's mass—that's 0.0000000000009 times. Assuming this mass value is approximately correct, Dactyl's gravitational field strength is about 0.009% of Earth's, so if you weigh 100 pounds on Earth, you'd weigh just 0.009 pounds on Dactyl.



Note: A plot of mass (vertical axis) against radius (horizontal axis), keeping surface gravity the same. Each one of these points changes the radius by a factor of 10, which to keep surface gravity the same means changing the mass by a factor of 100. All these numbers are written as a fraction of Earth's values. I've used something known as a logarithmic scale, which allows us to look at numbers of very different sizes easily.

So could a world like Asteroid B-612 exist? To see that, let's change the rules a bit: now we want to fix the gravitational field strength while shrinking the radius down to Little-Prince size. The graph above shows these relationships: an object 1/10 the size of Earth must have a mass 1/100 of Earth's to have the same gravitational attraction at its surface. That's a steep decrease: by the time we've reached Dactyl's size (about 10^{-4} Earth's radius), the mass required is 10^{-8} Earth's, or 100 millionth. While that's pretty small compared to Earth, it's also a *lot* more massive than Dactyl is in real life!

- Using the same ratios and relationships that is used on Dactyl, estimate how many times B612 radius is to Earth's radius.
- Estimate the B612 mass to Earth's mass.
- The implications are obvious: to make an asteroid with Earthlike gravity, you'd need to build it of rock far denser than anything found on Earth. Dactyl would need a

density 10,000 times Earth's density. How many more times is B612's density than Earth's density?

- d. Earth's density is 5.12 g/cm^3 . What is the density of B612?
- e. A white dwarf has a density of $1,000,000 \text{ g/cm}^3$. How does B612 compare to this?
- f. A neutron star has a density of $1 \times 10^{15} \text{ g/cm}^3$. How does B612 compare to this?
- g. Even as crazy as B-612 is, its gravity is not strong enough that we need general relativity. We can show this by calculating the hypothetical asteroid's Schwarzschild radius—the distance at which nothing can escape, not even light. If the Schwarzschild radius is bigger than the radius of the asteroid, then B-612 is actually a black hole. The formula for the Schwarzschild radius is remarkably simple:

$$R_{\text{Schwarzschild}} = 3 \text{ km} \frac{M}{M_{\text{Sun}}}$$

Meaning that an object with the same mass as our Sun has a Schwarzschild radius of 3 kilometers, something twice as massive will be 6 kilometers, and something half as massive will be 1.5 kilometers. Since B-612 has a tiny fraction of Earth's mass, which is in turn a tiny fraction of the Sun's mass, the Schwarzschild radius of B-612 is really minuscule: much smaller than the nucleus of an atom. Does this strengthen the fantasy concept of the Little Prince?

- h. One interesting final point about B-612 again relates to its tiny size. On Earth, we humans are but ants crawling across the vast globe: even the tallest person is short compared to the distance between the surface and center. That means the gravitational field is about the same on our heads as on our feet. The Moon, on the other hand, experiences a slightly stronger force on its near side than its far side, which over a period of billions of years has slowed its rotation until it now presents one face to Earth. This effect is known as *tidal locking*. The Moon also famously exerts a similar tidal force on Earth, which raises tides twice per day in the oceans. The Little Prince, standing about a meter tall on an asteroid itself only a meter in radius, will feel Earthlike gravity only on his feet. How much gravitational acceleration will his head experience, situated twice as far from the center of the asteroid, will experience? Could this be a problem for the little prince to function? Explain.
8. We've noted that B612 has volcanoes. To have volcanoes, you need a molten core. Does the size of an object is important to its core-moltenness? [Note: smaller objects radiate heat faster (proportionately) than larger objects, and larger objects have more nuclear fuel to work with.]

9. As far as molten cores go, there are three main sources of heat: formation heat, tidal forces, and radioactive decay.
- Formation heat is just the left over energy you get when you let a few trillion gigatons of stuff fall together. The formation heat of everything in the solar system was exhausted billions of years ago (except for Jupiter, which continues to slowly deflate and release heat. Essentially it's too fluffy). Would this be the source of B612's heat for a molten core? Explain.
 - Tidal forces only really apply to inner moons around gas giants. The tidal forces have to be huge in order to melt the core just by "massaging" the moon in question. Would this be the source of B612's heat for a molten core? Explain.
 - The most important thing for a liquid core is a supply of radioactive material. Given the amount of radioactive stuff left in the solar system today (it's been draining away for the last 5 billion years) an object needs to have a mass between about 1×10^{23} kg and 3×10^{23} kg (between 0.02 and 0.05 Earths, or around 70 million "Deimoses"), give or take. Would this be the source of B612's heat for a molten core? Explain.
10. The movie's mood is mostly adventurous and mysterious, with a philosophical overtone. At first the Little Prince does not reveal his identity, creating an initial sense of mystery. Then as the Little Prince recounts his travels, the mood becomes adventurous. As he questions the fox and the narrator, the mood becomes philosophic. At the end, when the Prince arranges to be bitten by the snake, the mood again becomes mysterious. Although he seems to die from the snake bite, the narrator cannot find the Prince's body when he looks for it the next morning. What do you think the narrator assumes happened to the Little Prince?
11. The need to have faith is another minor theme in the movie. The Little Prince arrives on the Earth during a spiritually troubled phase and stays until he has resolved his confusions. During his stay, he teaches the narrator the importance of having faith and belief. Many critics have called the Little Prince a Christ-figure. Explain why they think this and make this statement.

